Estimating with Finite Sums Practice

Name: _____ Date: _____

Question 1

estimate the area under the graph of the function

$f(x) = \frac{1}{x}$ between $x = 1$ and $x = 5$ decreasing on $[1, 5]$

lower sum with two rectangles of equal width :

lower sum with four rectangles of equal width :

upper sum with two rectangles of equal width :

upper sum with four rectangles of equal width :

midpoint rule (two rectangles) :

midpoint rule (four rectangles) :

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Question 2

estimate the average value of the function

$f(x) = \frac{1}{x}$ $[1, 9]$ four subintervals of equal length

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Question 1

estimate the area under the graph of the function

$f(x) = \frac{1}{x}$ between $x = 1$ and $x = 5$ decreasing on $[1, 5]$

lower sum with two rectangles of equal width :

$$\Delta x = \frac{5-1}{2} = 2 \longrightarrow x_i = 1 + i\Delta x = 1 + 2i \longrightarrow \sum_{i=1}^{2} \left(\frac{1}{x_i}\right) \cdot 2 = \left(\frac{1}{3} + \frac{1}{5}\right) \cdot 2 = \frac{16}{15}$$

lower sum with four rectangles of equal width :

$$\Delta x = \frac{5-1}{4} = 1 \longrightarrow x_i = 1 + i\Delta x = 1 + i \longrightarrow \sum_{i=1}^{4} \left(\frac{1}{x_i}\right) \cdot 1 = \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) \cdot 1 = \frac{77}{60}$$

upper sum with two rectangles of equal width :

$$\Delta x = \frac{5-1}{2} = 2 \longrightarrow x_i = 1 + i\Delta x = 1 + 2i \longrightarrow \sum_{i=0}^{1} \left(\frac{1}{x_i}\right) \cdot 2 = \left(1 + \frac{1}{3}\right) \cdot 2 = \frac{8}{3}$$

upper sum with four rectangles of equal width :

$$\Delta x = \frac{5-1}{4} = 1 \longrightarrow x_i = 1 + i\Delta x = 1 + i \longrightarrow \sum_{i=0}^{3} \left(\frac{1}{x_i}\right) \cdot 1 = \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \cdot 1 = \frac{25}{12}$$

midpoint rule (two rectangles) :

$$\Delta x = \frac{5-1}{2} = 2 \longrightarrow 2(f(2) + f(4)) \longrightarrow 2((1/2) + (1/4)) = 3/2$$

midpoint rule (four rectangles) :

$$\Delta x = \frac{5-1}{4} = 1 \longrightarrow 1(f(3/2) + f(5/2) + f(7/2) + f(9/2)) \longrightarrow 1((2/3) + (2/5) + (2/7) + (2/9)) = 496/315$$

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Question 2

estimate the average value of the function

$f(x) = \frac{1}{x}$ $[1,9]$ four subintervals of equal length

$[1,3]$	$[3,5]$	$[5,7]$	$[7,9]$	
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
2	4	6	8	midpoints
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	function

width of each rectangle : $\Delta x = 2$

$$2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right) = \frac{25}{12}$$

length of $[1,9]$: 8

$$\frac{\frac{25}{12}}{8} = \frac{25}{96}$$