Trigonometric Equations Practice

Name: _____ Date: _____

Question 1

Solve (calculator)

I. $\cos x = \frac{-1}{2}$

II. $\sin x = \frac{\sqrt{3}}{2}$

III. $\tan x = -\sqrt{3}$

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Question 2

Solve for solutions between [0,2π) (calculator)(solutions can also be found by graphing)

I. $2 \sin x + \sqrt{3} = 0$

II. $2 \sin x + \sqrt{2} = 0$

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III. $2\cos^2 x = 1$

IV. $2\cos x + \cos^2 x = 3$

V. $2\cos^2 x - \sqrt{3}\cos x = 0$

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VI. $2\sin x \cos x + 2\sin x - \cos x - 1 = 0$

VII. $5\cot(2x) = 5$

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VIII. $\tan x \sin x - \tan x = 0$

IX. $\sin(4x) - 2\sin(2x) = 0$

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Question 3

Solve for solutions between [0,2π) (use a graphing calculator to determine the solutions)

I. $\sin(2x)\cos x + \sin x = 0$

II. $2\sec x \tan x + 2\sec x + \tan x + 1 = 0$

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Question 4

Solve for solutions between [0,2π) (calculator)

I. $$\frac{\sin^2 x - 1}{\cos(\pi/2 - x) + 1} = \frac{\sqrt{2}}{2} - 1$$

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Question 5

Solve for solutions between $[0,2\pi)$ (use a graphing calculator to determine the solutions)

I. $x \sin x = 1$

II. $2\cos^2 x = x + 1$

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Name: **Key** Date: _____

Question 1

Solve (calculator)

I. $\cos x = \frac{-1}{2} \rightarrow x = \cos^{-1}\frac{-1}{2} \rightarrow \frac{2\pi}{3} + 2\pi \cdot n$

Solution 1:

Solution 2:

$2\pi - \frac{2\pi}{3} = \frac{4\pi}{3} \rightarrow \frac{4\pi}{3} + 2\pi \cdot n$

II. $\sin x = \frac{\sqrt{3}}{2} \rightarrow x = \sin^{-1}\frac{\sqrt{3}}{2} \rightarrow \frac{\pi}{3} + 2\pi \cdot n$

Solution 1:

Solution 2:

$\pi - \frac{\pi}{3} = \frac{2\pi}{3} \rightarrow \frac{2\pi}{3} + 2\pi \cdot n$

III. $\tan x = -\sqrt{3} \rightarrow x = \tan^{-1}(-\sqrt{3}) \rightarrow \frac{-\pi}{3} \rightarrow \pi + \frac{-\pi}{3} \rightarrow \frac{2\pi}{3} + \pi \cdot n$

negative solution

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Question 2

Solve for solutions between [0,2π) (calculator)(solutions can also be found by graphing)

I. $2\sin x + \sqrt{3} = 0 \rightarrow 2\sin x = -\sqrt{3} \rightarrow \sin x = -\sqrt{3}/2 \rightarrow x = \sin^{-1}(-\sqrt{3}/2) \rightarrow x = \frac{-\pi}{3}$

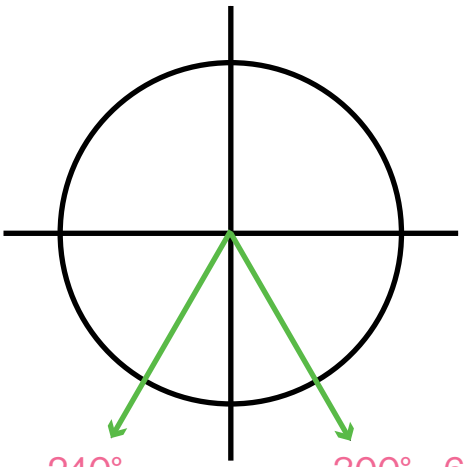
negative solution

Solution 1:

$\frac{-\pi}{3} + 2\pi \rightarrow \frac{5\pi}{3}$

Solution 2:

$240° \cdot \frac{\pi}{180°} \rightarrow \frac{4\pi}{3}$



$\sin(-60°) = -\sqrt{3}/2$

$\sin(300°) = -\sqrt{3}/2$

$\sin(240°) = -\sqrt{3}/2$

240°
sin is negative Q3

300°, -60°
sin is negative Q4

solutions are in Q3 and Q4

II. $2\sin x + \sqrt{2} = 0 \rightarrow 2\sin x = -\sqrt{2} \rightarrow \sin x = -\sqrt{2}/2 \rightarrow x = \sin^{-1}(-\sqrt{2}/2) \rightarrow x = \frac{-\pi}{4}$

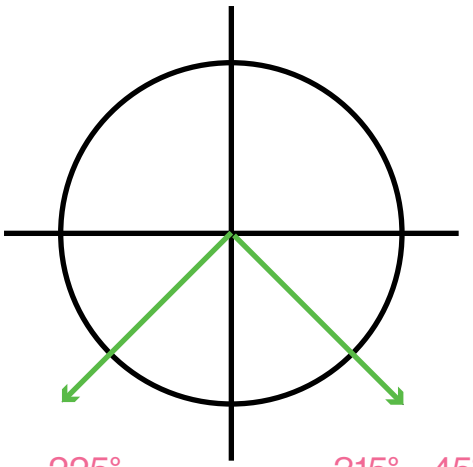
negative solution

Solution 1:

$\frac{-\pi}{4} + 2\pi \rightarrow \frac{7\pi}{4}$

Solution 2:

$225° \cdot \frac{\pi}{180°} \rightarrow \frac{5\pi}{4}$



$\sin(-45°) = -\sqrt{2}/2$

$\sin(315°) = -\sqrt{2}/2$

$\sin(225°) = -\sqrt{2}/2$

225°
sin is negative Q3

315°, -45°
sin is negative Q4

solutions are in Q3 and Q4

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III. $2\cos^2 x = 1 \rightarrow \cos^2 x = 1/2 \rightarrow \cos x = \pm\sqrt{\frac{1}{2}} \rightarrow \cos x = \pm \frac{1}{\sqrt{2}}$

$\cos x = \frac{1}{\sqrt{2}} \rightarrow x = \cos^{-1}(1/\sqrt{2}) \rightarrow$ Solution 1: $\frac{\pi}{4}$ Solution 2: $\frac{7\pi}{4}$

$\cos x = \frac{-1}{\sqrt{2}} \rightarrow x = \cos^{-1}(-1/\sqrt{2}) \rightarrow$ Solution 3: $\frac{3\pi}{4}$ Solution 4: $\frac{5\pi}{4}$

IV. $2\cos x + \cos^2 x = 3 \rightarrow 2\cos x + \cos^2 x - 3 = 0 \rightarrow 2\cos x + \cos^2 x + -3 = 0$

$\downarrow$

$\cos^2 x + 2\cos x + -3 = 0$

$\cos x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} \rightarrow$

$\cos x = 1 \rightarrow x = \cos^{-1} 1 \rightarrow x = 0$

$\cos x = -3 \rightarrow x = \cos^{-1} -3 \rightarrow$ no solution

V. $2\cos^2 x - \sqrt{3}\cos x = 0 \rightarrow \cos x\,(2\cos x - \sqrt{3}) = 0$

$\cos x = 0 \rightarrow x = \cos^{-1} 0 \rightarrow x = \frac{\pi}{2}$

$2\cos x - \sqrt{3} = 0 \rightarrow 2\cos x = \sqrt{3} \rightarrow \cos x = \sqrt{3}/2 \rightarrow x = \cos^{-1}\sqrt{3}/2 \rightarrow x = \frac{\pi}{6}$

Solution 1: $\frac{\pi}{2}$ Solution 2: $\frac{3\pi}{2}$

Solution 3: $\frac{\pi}{6}$ Solution 4: $\frac{11\pi}{6}$

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VI. $2\sin x \cos x + 2\sin x - \cos x - 1 = 0 \rightarrow 2\sin x(\cos x + 1) - (\cos x + 1) = 0 \rightarrow (2\sin x - 1)(\cos x + 1) = 0$

$2\sin x - 1 = 0 \rightarrow 2\sin x = 1 \rightarrow \sin x = 1/2 \rightarrow x = \sin^{-1} 1/2 \rightarrow x = \frac{\pi}{6}$

$\cos x + 1 = 0 \rightarrow \cos x = -1 \rightarrow x = \cos^{-1} -1 \rightarrow x = \pi$

Solution 1: $\frac{\pi}{6}$

Solution 2: $\frac{5\pi}{6}$

Solution 3: π

VII. $5\cot(2x) = 5 \rightarrow \cot(2x) = 1 \rightarrow 2x = \cot^{-1} 1 \rightarrow 2x = \tan^{-1}(1/1) \rightarrow 2x = \frac{\pi}{4}$

Solution 1: $\frac{\pi}{8}$

Solution 2: $\frac{5\pi}{8}$

Solution 3: $\frac{9\pi}{8}$

Solution 4: $\frac{13\pi}{8}$

$\downarrow$

$x = \frac{\pi}{8}$

$\downarrow$

$\frac{\pi}{8} + \frac{\pi \cdot n}{2}$ period

$\frac{\pi}{8} + \frac{\pi \cdot (1)}{2} \rightarrow \frac{5\pi}{8}$ $\quad \frac{\pi}{8} + \frac{\pi \cdot (2)}{2} \rightarrow \frac{9\pi}{8}$ $\quad \frac{\pi}{8} + \frac{\pi \cdot (3)}{2} \rightarrow \frac{13\pi}{8}$

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VIII. $\tan x \sin x - \tan x = 0 \rightarrow \tan x (\sin x - 1) = 0$

$\tan x = 0 \rightarrow x = \tan^{-1} 0 \rightarrow x = 0$

$\sin x - 1 = 0 \rightarrow \sin x = 1 \rightarrow x = \sin^{-1} 1 \rightarrow x = \frac{\pi}{2}$ not true

Solution 1: 0

Solution 2: π

IX. $\sin(4x) - 2\sin(2x) = 0 \rightarrow \sin[2(2x)] - 2\sin(2x) = 0 \rightarrow 2\sin(2x)\cos(2x) - 2\sin(2x) = 0$

double-angle identities

$\downarrow$

$2\sin(2x)(\cos(2x) - 1) = 0$

$2\sin(2x) = 0 \rightarrow \sin(2x) = 0 \rightarrow 2x = \sin^{-1} 0 \rightarrow 2x = 0 \rightarrow x = 0$

$\downarrow$

Solution 1: 0

Solution 2: $\frac{\pi}{2}$

$0 + \frac{\pi \cdot n}{2}$ period

Solution 3: π

Solution 4: $\frac{3\pi}{2}$

$\cos(2x) - 1 = 0 \rightarrow \cos(2x) = 1 \rightarrow 2x = \cos^{-1} 1 \rightarrow 2x = 0 \rightarrow x = 0$

$\downarrow$

Solution 5: 0

same as solution 1

Solution 6: π

same as solution 3

$0 + \pi \cdot n$ period

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Question 3

Solve for solutions between [0,2π) (use a graphing calculator to determine the solutions)

I. $\sin(2x)\cos x + \sin x = 0$

Solution 1: 0 Solution 2: π

II. $2\sec x \tan x + 2\sec x + \tan x + 1 = 0 \rightarrow 2\sec x(\tan x + 1) + (\tan x + 1) = 0$

$\downarrow$

$(2\sec x + 1)(\tan x + 1) = 0$

$2\sec x + 1 = 0 \rightarrow$ no solution

$\tan x + 1 = 0$

Solution 1: $\frac{3\pi}{4}$ Solution 2: $\frac{7\pi}{4}$

III. $\sec^2 x - 2\tan^2 x = 0$

Solution 1: $\frac{\pi}{4}$ Solution 2: $\frac{3\pi}{4}$

Solution 3: $\frac{5\pi}{4}$ Solution 4: $\frac{7\pi}{4}$

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Trigonometric Equations Practice

Question 4

Solve for solutions between [0,2π) (calculator)

I. $$\frac{\sin^2 x - 1}{\cos(\pi/2 - x) + 1} = \frac{\sqrt{2}}{2} - 1$$

↓

$$\frac{\sin^2 x - 1}{\sin x + 1} = \frac{\sqrt{2}}{2} - 1$$

cofunction identities

↓

$$\frac{(\sin x - 1)(\sin x + 1)}{\sin x + 1} = \frac{\sqrt{2}}{2} - 1$$

↓

$$\sin x - 1 = \frac{\sqrt{2}}{2} - 1$$

↓

$$\sin x = \frac{\sqrt{2}}{2}$$

↓

$$x = \sin^{-1}\frac{\sqrt{2}}{2} \rightarrow$$

Solution 1: $\frac{\pi}{4}$ Solution 2: $\frac{3\pi}{4}$

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Question 5

Solve for solutions between [0,2π) (use a graphing calculator to determine the solutions)

I. $x \sin x = 1 \rightarrow x \sin x - 1 = 0$

Solution 1:
1.114

Solution 2:
2.773

II. $2\cos^2 x = x + 1 \rightarrow 2\cos^2 x - x - 1 = 0$

Solution 1:
0.515

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