

# Difference Quotients Practice

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Question 1

Simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

I.  $f(x) = 6x + 2$

II.  $f(x) = -\frac{1}{4x}$

# Difference Quotients Practice

$$\frac{f(x+h) - f(x)}{h}$$

III.  $f(x) = x^2 - 3$

IV.  $f(x) = 2|x| + 3x$

V.  $f(x) = x^3$

# Difference Quotients Practice

$$\frac{f(x+h) - f(x)}{h}$$

$$\text{VI. } f(x) = \frac{x-4}{x+3}$$

# Difference Quotients Practice

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## Question 1

Simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

I.  $f(x) = 6x + 2$

$$f(x+h) \longrightarrow 6(x+h) + 2 \longrightarrow 6x + 6h + 2$$

$$\frac{6x + 6h + 2 - (6x + 2)}{h} \longrightarrow \frac{\cancel{6x} + 6h + \cancel{2} - \cancel{6x} - \cancel{2}}{h} \longrightarrow \frac{\cancel{6h}}{h} \longrightarrow 6$$

II.  $f(x) = -\frac{1}{4x}$

$$f(x+h) \longrightarrow -\frac{1}{4(x+h)}$$

$$\frac{-\frac{1}{4(x+h)} - \left(-\frac{1}{4x}\right)}{h} \longrightarrow \frac{-\frac{1}{4(x+h)} + \frac{1}{4x}}{h} \longrightarrow \frac{-\frac{1}{4(x+h)} \cdot \frac{x}{x} + \frac{1}{4x} \cdot \frac{x+h}{x+h}}{h}$$

$$\frac{\frac{\cancel{h}}{4x(x+h)} \cdot \frac{1}{\cancel{h}}}{h} \longleftarrow \frac{\frac{h}{4x(x+h)}}{h} \longleftarrow \frac{\frac{-x + x + h}{4x(x+h)}}{h} \longleftarrow \frac{-\frac{x}{4x(x+h)} + \frac{x+h}{4x(x+h)}}{h}$$

$$\downarrow$$

$$\frac{1}{4x(x+h)}$$

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$$\frac{f(x+h) - f(x)}{h}$$

III.  $f(x) = x^2 - 3$

$$f(x+h) \longrightarrow (x+h)^2 - 3$$

$$\begin{aligned} \frac{(x+h)^2 - 3 - (x^2 - 3)}{h} &\longrightarrow \frac{\cancel{x^2} + 2\cancel{x}h + \cancel{h^2} - 3 - \cancel{x^2} + 3}{h} \\ &\quad \downarrow \\ 2\cancel{x} + h &\longleftarrow \frac{\cancel{h}(2\cancel{x} + h)}{\cancel{h}} \longleftarrow \frac{2\cancel{x}h + h^2}{h} \end{aligned}$$

IV.  $f(x) = 2|x| + 3x$

$$f(x+h) \longrightarrow 2|x+h| + 3(x+h)$$

$$\frac{2|x+h| + 3(x+h) - (2|x| + 3x)}{h} \longrightarrow \frac{2|x+h| + 3\cancel{x} + 3h - 2|x| - 3\cancel{x}}{h} \longrightarrow \frac{2|x+h| + 3h - 2|x|}{h}$$

V.  $f(x) = x^3$

$$f(x+h) \longrightarrow (x+h)^3 \longrightarrow x^3 + 3x^2h + 3xh^2 + h^3$$

$$\begin{aligned} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - (x^3)}{h} &\longrightarrow \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h} \longrightarrow \frac{3x^2h + 3xh^2 + h^3}{h} \\ &\quad \downarrow \\ 3x^2 + 3xh + h^2 &\longleftarrow \frac{\cancel{h}(3\cancel{x^2} + 3\cancel{x}h + h^2)}{\cancel{h}} \end{aligned}$$

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$$\frac{f(x+h) - f(x)}{h}$$

$$\text{VI. } f(x) = \frac{x-4}{x+3}$$

$$f(x+h) \rightarrow \frac{x+h-4}{x+h+3}$$

$$\frac{\frac{x+h-4}{x+h+3} - \frac{x-4}{x+3}}{h} \rightarrow \frac{\frac{x+h-4}{x+h+3} - \frac{x-4}{x+3}}{h} \cdot \frac{(x+h+3)(x+3)}{(x+h+3)(x+3)}$$

$$\downarrow$$

$$\frac{(x+h-4)(x+3) - (x-4)(x+h+3)}{h(x+h+3)(x+3)}$$



$$\frac{x^2 + hx + 3h - x - 12 - (hx + x^2 - 4h - x - 12)}{h(x+h+3)(x+3)} \rightarrow \frac{\cancel{x^2} + \cancel{hx} + 3h - \cancel{x} - 12 - \cancel{hx} - \cancel{x^2} + 4h + \cancel{x} + 12}{h(x+h+3)(x+3)}$$



$$\frac{7}{(x+h+3)(x+3)} \leftarrow \frac{\cancel{7h}}{\cancel{h}(x+h+3)(x+3)}$$