

Polynomials Practice

Name: _____

Date: _____

Question 1

Determine the zeros of the polynomial

1. $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$

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II. $f(x) = 2x^3 - 3x^2 + x + 6$

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III. $f(x) = (x + 5)^3(x - 4)(x + 1)^2$ Zeros:

IV. $f(x) = (x^2 - 9)^3$ Zeros:

V. $f(x) = x^3(x - 1)^2(x + 4)$ Zeros:

VI. $f(x) = -8(x - 3)^2(x + 4)^3x^4$ Zeros:

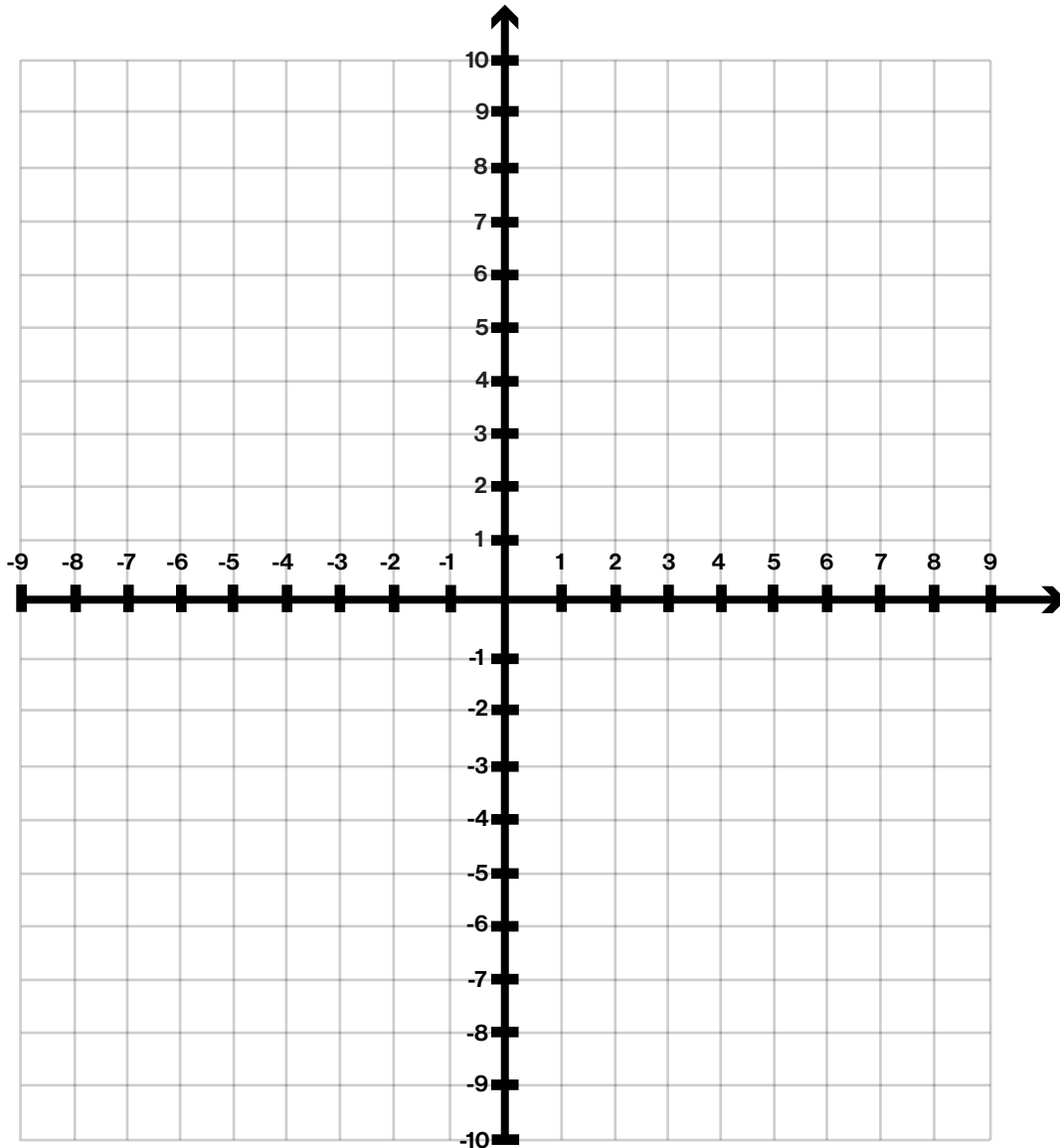
VII. $f(x) = (x^2 - 5x + 6)^2$ Zeros:

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Question 2

Graph (graphing calculator)

I. $f(x) = x^3 - 3x - 1$



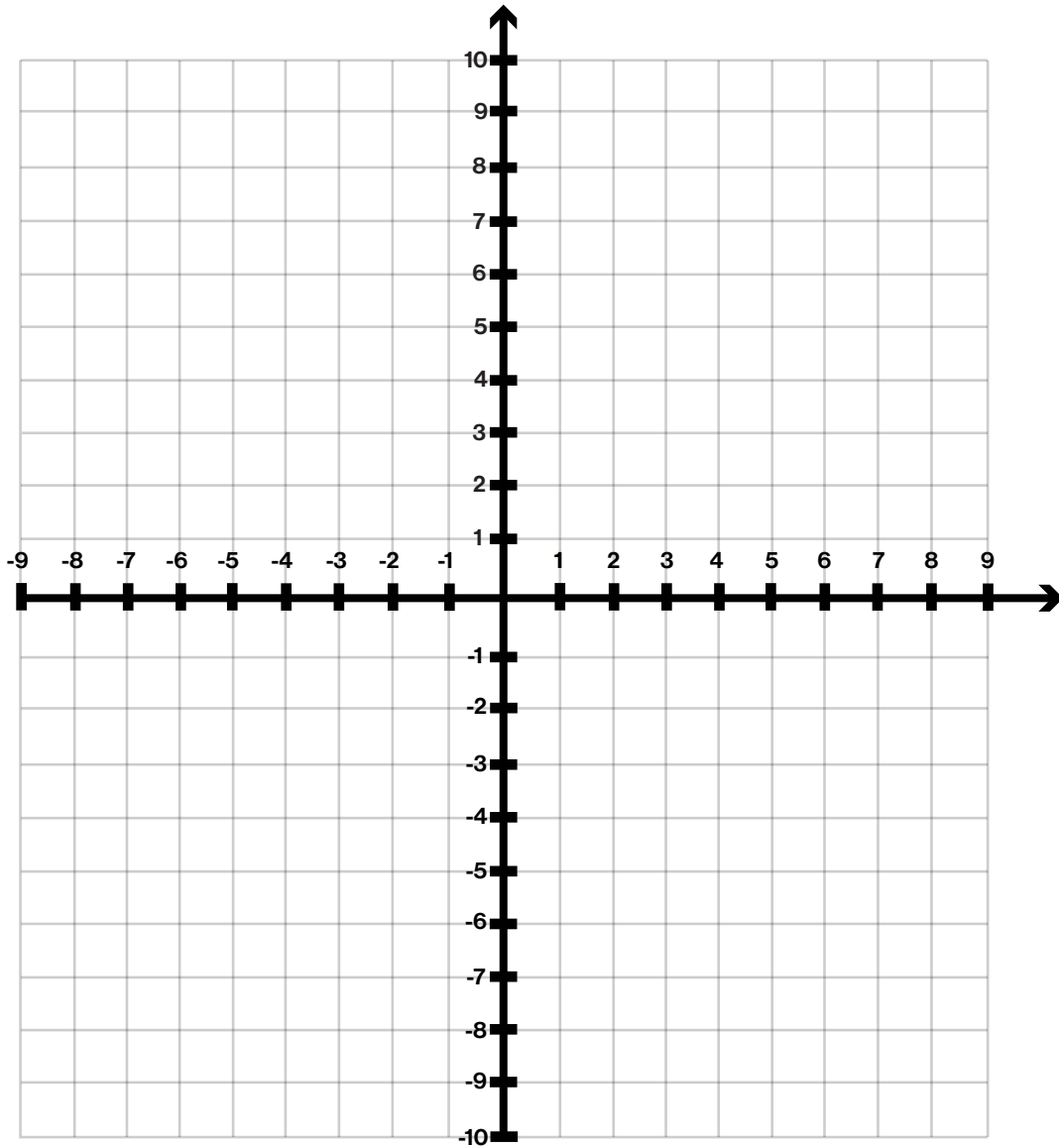
relative maxima :

relative minima :

range :

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II. $f(x) = x^3 - 1.2x + 1$



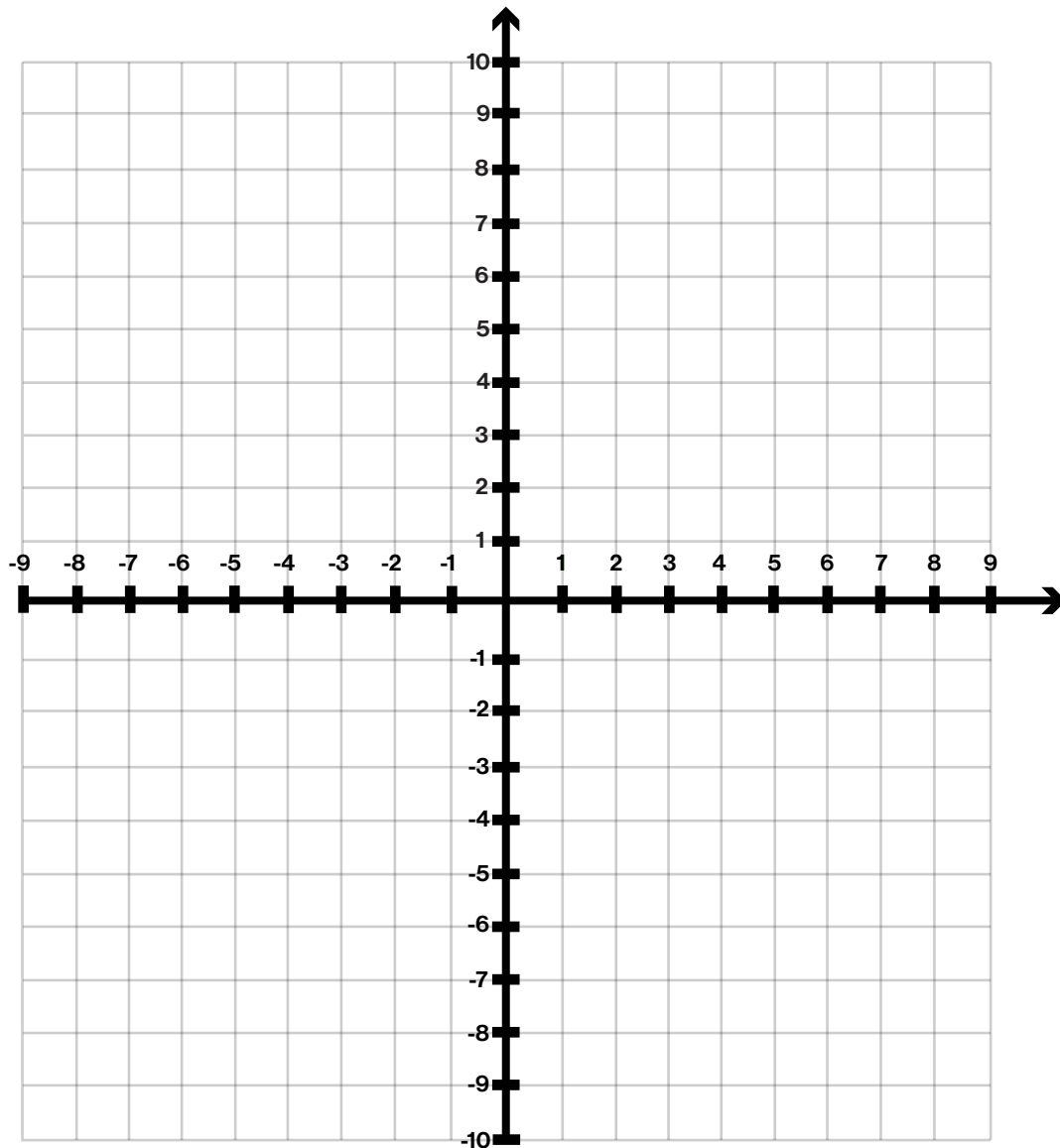
relative maxima :

relative minima :

range :

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III. $f(x) = x^6 - 3.8$



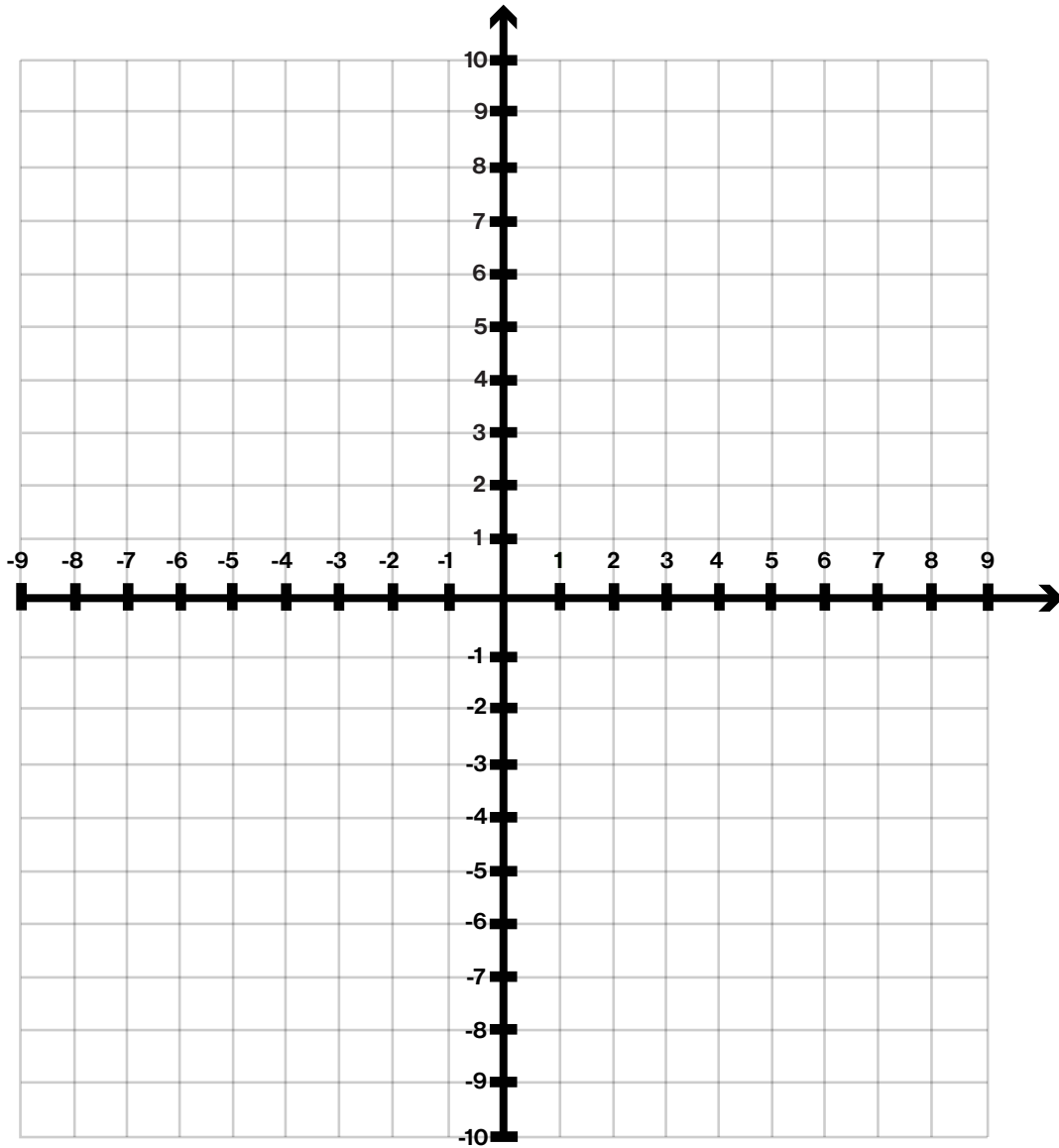
relative maxima :

relative minima :

range :

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IV. $f(x) = -x^2 + x$



relative maxima :

relative minima :

range :

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Question 3

Find the corresponding polynomial of degree 3

I. Zeros: $-2, 3, 5$

II. Zeros: $2, i, -i$

III. Zeros: $-5, \sqrt{3}, -\sqrt{3}$

IV. Zeros: $-2, 1 - \sqrt{3}, 1 + \sqrt{3}$

V. Zeros: $-1, 1 - 4i, 1 + 4i$

VI. Zeros: $-3, 0, 0.5$

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Name: _____ **Key** _____

Date: _____

Question 1

Determine the zeros of the polynomial

I. $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$

$$f(x) = x^4 + -2x^3 + -x^2 + 6x + -6 \rightarrow f(x) = -2x^3 + 6x + x^4 + -x^2 + -6$$

A

$$\begin{array}{c} -2x^3 + 6x \\ \downarrow \\ -2x(x^2 - 3) \end{array}$$

B

$$\begin{array}{c} x^4 + -x^2 + -6 \\ \downarrow \\ x^4 - x^2 - 6 \\ \downarrow \\ a^2 - a - 6 \\ \downarrow \\ (a - 3)(a + 2) \\ \downarrow \\ (x^2 - 3)(x^2 + 2) \end{array}$$

$a^2 = x^4$
 $a = x^2$

Combine:

$$-2x(x^2 - 3) + (x^2 - 3)(x^2 + 2) \rightarrow (x^2 - 3)(-2x + x^2 + 2) \rightarrow (x^2 - 3)(x^2 - 2x + 2)$$

$$x^2 - 3 = 0$$

$$x^2 - 2x + 2 = 0$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

$$\frac{-(-2) + \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \rightarrow \frac{2 + \sqrt{-4}}{2} \rightarrow \frac{2 + 2i}{2} \rightarrow \frac{2(1 + i)}{2}$$

$$\downarrow$$

$$1 + i$$

$$\frac{-(-2) - \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} \rightarrow \frac{2 - \sqrt{-4}}{2} \rightarrow \frac{2 - 2i}{2} \rightarrow \frac{2(1 - i)}{2}$$

$$\downarrow$$

$$1 - i$$

Zeros: $\pm\sqrt{3}$ and $1 \pm i$

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II. $f(x) = 2x^3 - 3x^2 + x + 6$

Factors of Constant:

$$p = \pm 1, \pm 2, \pm 3, \pm 6$$

Factors of Leading Coefficient:

$$q = \pm 1, \pm 2$$

Possible Roots:

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 0.5, \pm 1.5$$

$$2(-1)^3 - 3(-1)^2 + (-1) + 6 = 0 \quad \checkmark$$

$$\begin{array}{r}
 2x^2 - 5x + 6 \\
 x+1 \overline{) 2x^3 - 3x^2 + x + 6} \\
 \underline{2x^3 + 2x^2} \\
 -5x^2 + x \\
 \underline{-5x^2 - 5x} \\
 6x + 6 \\
 \underline{6x + 6} \\
 0
 \end{array}$$

$$1. \frac{2x^3}{x} \rightarrow 2x^2 \rightarrow 2x^2(x+1) \rightarrow 2x^3 + 2x^2$$

$$2. \frac{-5x^2}{x} \rightarrow -5x \rightarrow -5x(x+1) \rightarrow -5x^2 - 5x$$

$$3. \frac{6x}{x} \rightarrow 6 \rightarrow 6(x+1) \rightarrow 6x + 6$$

$$2x^2 - 5x + 6 = 0$$

$$\frac{-(-5) + \sqrt{(-5)^2 - 4(2)(6)}}{2(2)} \rightarrow \frac{5 + \sqrt{-23}}{4} \rightarrow \frac{5 + i\sqrt{23}}{4}$$

$$\frac{-(-5) - \sqrt{(-5)^2 - 4(2)(6)}}{2(2)} \rightarrow \frac{5 - \sqrt{-23}}{4} \rightarrow \frac{5 - i\sqrt{23}}{4}$$

Zeros: -1 and $\frac{5 \pm i\sqrt{23}}{4}$

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III. $f(x) = (x+5)^3(x-4)(x+1)^2$ Zeros: $-5, 4, -1$

$$x+5=0 \rightarrow x=-5$$

$$x-4=0 \rightarrow x=4$$

$$x+1=0 \rightarrow x=-1$$

IV. $f(x) = (x^2-9)^3$

$$f(x) = (x+3)^3(x-3)^3 \quad \text{Zeros: } -3, 3$$

$$x+3=0 \rightarrow x=-3$$

$$x-3=0 \rightarrow x=3$$

V. $f(x) = x^3(x-1)^2(x+4)$ Zeros: $0, 1, -4$

$$x^3=0 \rightarrow x=0$$

$$x-1=0 \rightarrow x=1$$

$$x+4=0 \rightarrow x=-4$$

VI. $f(x) = -8(x-3)^2(x+4)^3x^4$ Zeros: $3, -4, 0$

$$x-3=0 \rightarrow x=3$$

$$x+4=0 \rightarrow x=-4$$

$$x^4=0 \rightarrow x=0$$

VII. $f(x) = (x^2-5x+6)^2$ Zeros: $3, 2$

$$(x^2-5x+6)^2 \rightarrow (x-3)^2(x-2)^2$$

$$x-3=0 \rightarrow x=3$$

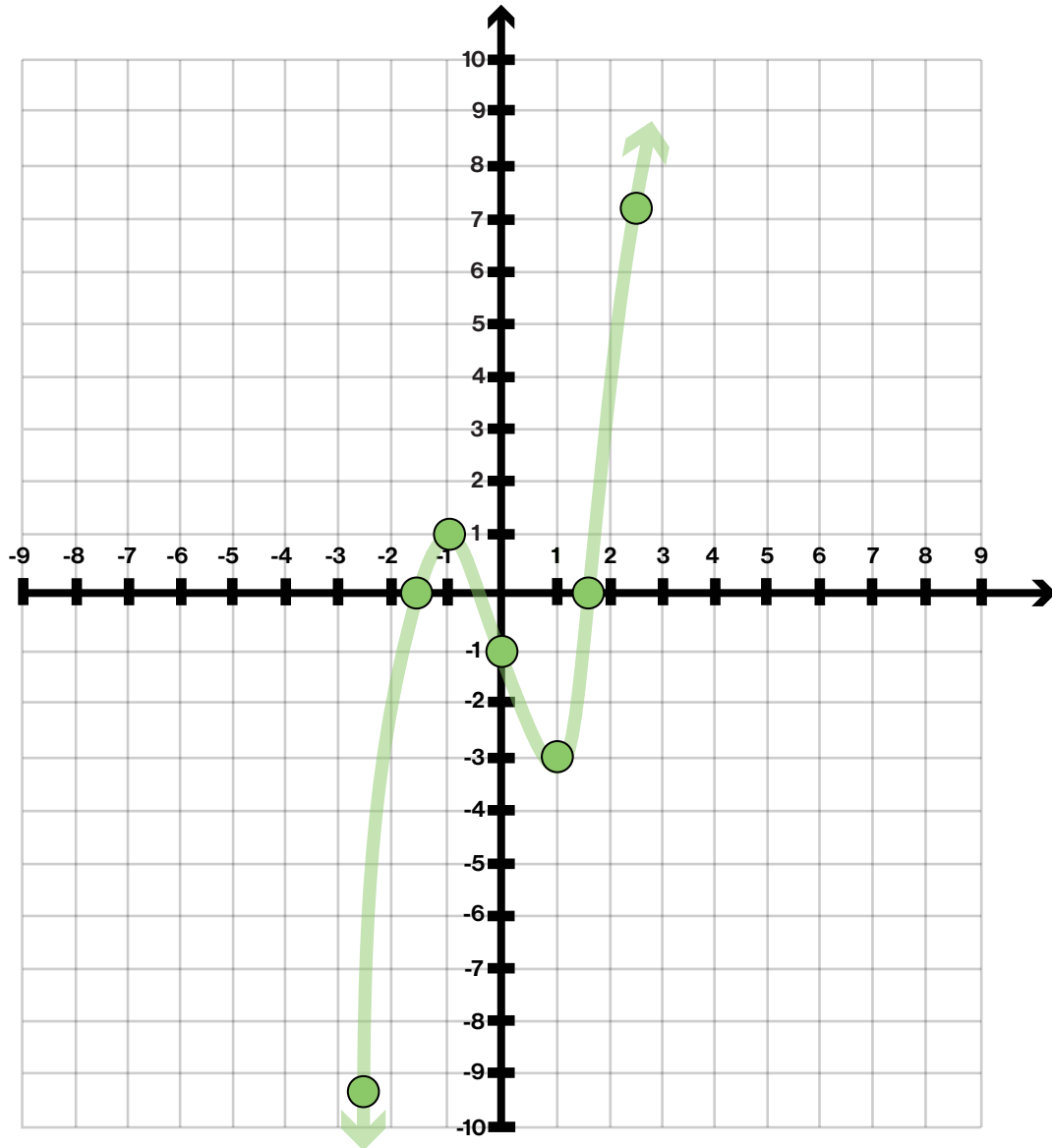
$$x-2=0 \rightarrow x=2$$

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Question 2

Graph (graphing calculator)

1. $f(x) = x^3 - 3x - 1$



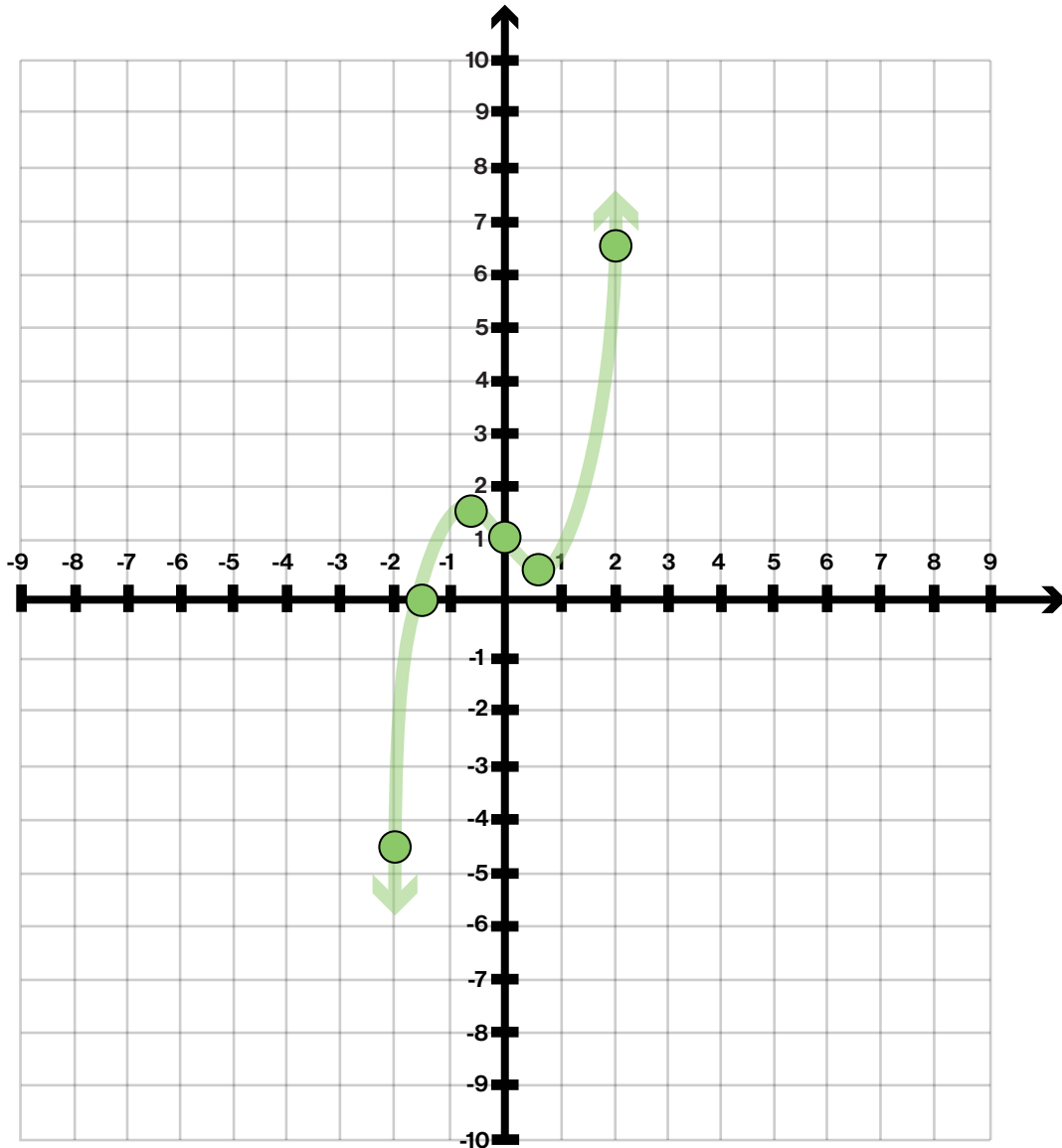
relative maxima: $(-1, 1) \rightarrow 1$

relative minima: $(1, -3) \rightarrow -3$

range: $(-\infty, \infty)$

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II. $f(x) = x^3 - 1.2x + 1$



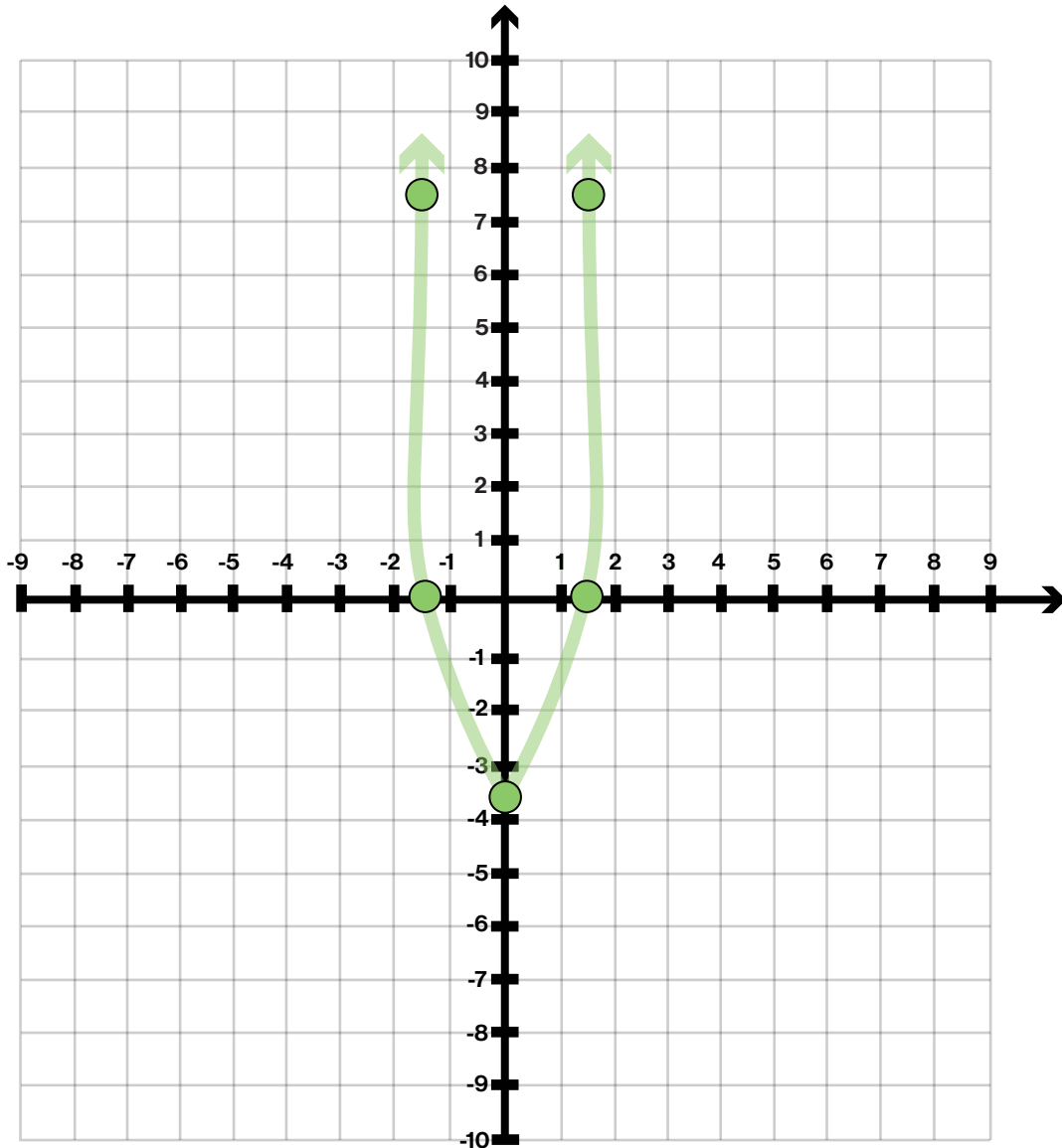
relative maxima: $(-0.632, 1.506) \rightarrow 1.506$

relative minima: $(0.632, 0.494) \rightarrow 0.494$

range: $(-\infty, \infty)$

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III. $f(x) = x^6 - 3.8$



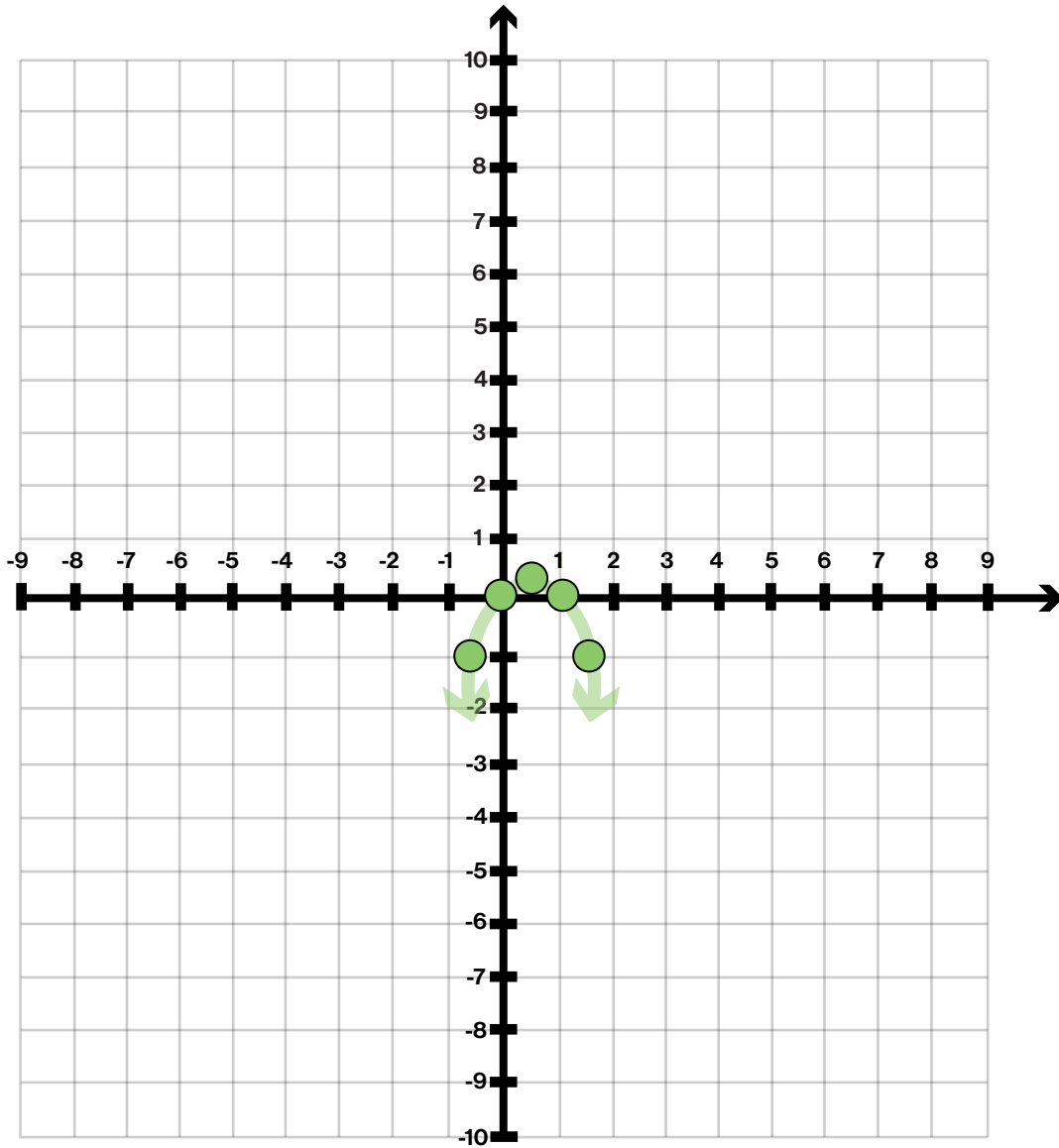
relative maxima : N/A

relative minima : $(0, -3.8) \rightarrow -3.8$

range : $[-3.8, \infty)$

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IV. $f(x) = -x^2 + x$



relative maxima : $(0.5, 0.25) \rightarrow 0.25$

relative minima : N/A

range : $(-\infty, 0.25]$

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Question 3

Find the corresponding polynomial of degree 3

I. Zeros: $-2, 3, 5$

$$f(x) = (x+2)(x-3)(x-5) \rightarrow f(x) = (x+2)(x^2-8x+15) \rightarrow f(x) = x^3-6x^2-x+30$$

II. Zeros: $2, i, -i$

$$f(x) = (x-2)(x-i)(x+i) \rightarrow f(x) = (x-2)(x^2+1) \rightarrow f(x) = x^3-2x^2+x-2$$

III. Zeros: $-5, \sqrt{3}, -\sqrt{3}$

$$f(x) = (x+5)(x-\sqrt{3})(x+\sqrt{3}) \rightarrow f(x) = (x+5)(x^2-3) \rightarrow f(x) = x^3+5x^2-3x-15$$

IV. Zeros: $-2, 1-\sqrt{3}, 1+\sqrt{3}$

$$f(x) = (x+2)[x-(1-\sqrt{3})][x-(1+\sqrt{3})] \rightarrow f(x) = (x+2)[(x-1)+\sqrt{3}][(x-1)-\sqrt{3}]$$

↓

$$f(x) = (x+2)[(x-1)^2-(\sqrt{3})^2]$$

↓

$$f(x) = (x+2)(x^2-2x-2)$$

↓

$$f(x) = x^3-6x-4$$

V. Zeros: $-1, 1-4i, 1+4i$

$$f(x) = (x+1)[x-(1-4i)][x-(1+4i)] \rightarrow f(x) = (x+1)[(x-1)+4i][(x-1)-4i]$$

↓

$$f(x) = (x+1)[(x-1)^2-(4i)^2]$$

↓

$$f(x) = (x+1)(x^2-2x+17)$$

↓

$$f(x) = x^3-x^2+15x+17$$

VI. Zeros: $-3, 0, 0.5$

$$f(x) = (x+3)(x)(x-0.5) \rightarrow f(x) = (x+3)(x^2-0.5x) \rightarrow f(x) = x^3+2.5x^2-1.5x$$